

Brans-Dicke Cosmology in an anisotropic model when Velocity of Light Varies

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In this paper, we have studied Brans-Dicke Cosmology in anisotropic Kantowski-Sachs space-time model; considering variation of the velocity of light. We have addressed the flatness problem considering both the cases namely, (i) when the Brans-Dicke scale field ϕ is constant (ii) when ϕ varies, specially for radiation dominated era perturbatively and non-perturbatively and asymptotic behaviour have been studied.

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I. INTRODUCTION

In recent years theories with varying speed of light (VSL) has been attracted considerable attention [1-10] due to its ability to solve the so-called cosmological puzzles - the horizon, flatness and Lambda problems of big-bang cosmology. All previous attempts to overcome these difficulties invoke the basic inflationary form, where the observable universe experiences a period of ‘superluminal’ expansion. Here, one has to modify the matter content of the universe such that ordinary Einstein gravity becomes repulsive and halts the exponential expansion. One can however resolve to theories with varying speed of light as the alternative method to solve the above mentioned cosmological puzzles. Here, instead of changing the matter content of the universe, one has to change the speed of light in the early universe. The universe is assumed to be radiation dominated in the early stage and the matter content is assumed to be the same as in the standard big-bang (SBB) model. So the geometry and expansion factor of the universe go in accordance with the SBB model. Here for a free falling observer associated with the cosmic expansion, the local speed of light will decelerate from a very large value to its current value.

So far some works have been done with VSL by Magueijo and co-workers [1-5] and others [6-10] specially in isotropic cosmological models (which also include BD-theory). Very recently, Magueijo has investigated [11], the possibility of black holes formation by studying spherically symmetric solutions with VSL. In this paper, we will investigate in details the anisotropic Kantowski-Sachs (KS) model with VSL theory. The paper is organized as follows: section II deals with the basic equations in BD-theory for KS model with varying speed of light. Perturbative solution to the flatness problem will be discussed in section III, while non-perturbative study of this problem will be discussed in section IV for radiation era only and the exact solution to the flatness problem is presented in section V. In section VI, we will study solutions to the quasi-flatness problem and examine its asymptotic behaviour. The lambda and quasi-lambda problems and their asymptotic behaviours will be investigated in section VII. The paper concludes with a short discussion in section VIII.

II. THE BASIC EQUATIONS

For anisotropic KS model with metric ansatz

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$$ds^2 = -c^2 dt^2 + a^2(t) dr^2 + b^2(t) d\Omega_2^2 ,$$

the BD field equations with varying speed of light are

$$\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} = -\frac{8\pi}{(3+2\omega)\phi} \left[(2+\omega)\rho + 3(1+\omega)\frac{p}{c^2} \right] - \omega \left(\frac{\dot{\phi}}{\phi} \right)^2 - \frac{\ddot{\phi}}{\phi} \quad (1)$$

$$\left(\frac{\dot{b}}{b} \right)^2 + 2\frac{\dot{a}\dot{b}}{ab} = \frac{8\pi\rho}{\phi} - \frac{c^2}{b^2} - \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) \frac{\dot{\phi}}{\phi} + \frac{\omega}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 \quad (2)$$

and the wave equation is

$$\ddot{\phi} + \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) \dot{\phi} = \frac{8\pi}{3+2\omega} \left(\rho - \frac{3p}{c^2} \right) \quad (3)$$

Here the velocity of light c is an arbitrary function of time, ω is the BD coupling parameter and the BD scalar field is $\phi = \frac{1}{G}$. From the above field equations we have the “non-conservation” equation

$$\dot{\rho} + \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) \left(\rho + \frac{p}{c^2} \right) = \frac{cc}{4\pi b^2} \phi \quad (4)$$

Now $p = \frac{1}{3}\rho c^2$ is the equation of state in the radiation era for which the general solution ϕ is (from equation (3))

$$\phi = \phi_0 + \alpha \int \frac{dt}{ab^2}. \quad (5)$$

The above field equations in BD-theory have been formulated in Jordan frame. To switch over to Einstein frame we shall have to make the following transformations

$$\begin{aligned} \hat{d}t &= \sqrt{G\phi} dt, \quad \hat{a} = \sqrt{G\phi} a, \quad \hat{b} = \sqrt{G\phi} b, \quad \sigma = \left(\omega + \frac{3}{2} \right)^{1/2} \ln(G\phi), \quad \hat{\rho} = (G\phi)^{-2} \rho, \\ \hat{p} &= (G\phi)^{-2} p, \end{aligned} \quad (6)$$

and the above field equations become (c is treated as constant)

$$\frac{\hat{a}''}{\hat{a}} + 2\frac{\hat{b}''}{\hat{b}} = -4\pi G \left(\hat{\rho} + \frac{3\hat{p}}{c^2} \right) - \sigma'^2, \quad (7)$$

$$\left(\frac{\hat{b}'}{\hat{b}} \right)^2 + 2\frac{\hat{a}'\hat{b}'}{\hat{a}\hat{b}} + \frac{c^2}{\hat{b}^2} = 8\pi G \hat{\rho} + \frac{\hat{\sigma}'^2}{2}, \quad (8)$$

and

$$\sigma'' + \left(\frac{\hat{a}'}{\hat{a}} + 2\frac{\hat{b}'}{\hat{b}} \right) \sigma' = \frac{8\pi G}{\sqrt{6+4\omega}} \left(\hat{\rho} - \frac{3\hat{p}}{c^2} \right) \quad (9)$$

with the prime “ ‘ ” and “ . ” represent the differentiation with respect to \hat{t} and t respectively.

One can interpret these field equations as standard KS equations with constant G and a scalar field σ is added to the normal matter . The scalar field behaves like a “stiff” perfect fluid with equation of state

$$\hat{p}_\sigma = \hat{\rho}_\sigma = \frac{\sigma'^2}{16\pi G} . \quad (10)$$

If the velocity of light is constant , then in Einstein frame total stress-energy tensor is conserved but there is an exchange of energy between the scalar field and normal matter according to the following equation

$$\hat{\rho}' + \left(\frac{\hat{a}'}{\hat{a}} + 2\frac{\hat{b}'}{\hat{b}} \right) \left(\hat{\rho} + \frac{\hat{p}}{c^2} \right) = - \left[\hat{\rho}'_\sigma + \left(\frac{\hat{a}'}{\hat{a}} + 2\frac{\hat{b}'}{\hat{b}} \right) \left(\hat{\rho}_\sigma + \frac{\hat{p}_\sigma}{c^2} \right) \right] = - \frac{\sigma'}{\sqrt{6+4\omega}} \left(\hat{\rho} - \frac{3\hat{p}}{c^2} \right) \quad (11)$$

On the other hand , if the velocity of light varies then we have two separate “non-conservation” equations

$$\hat{\rho}' + \left(\frac{\hat{a}'}{\hat{a}} + 2\frac{\hat{b}'}{\hat{b}} \right) \left(\hat{\rho} + \frac{\hat{p}}{c^2} \right) = - \frac{\sigma'}{\sqrt{6+4\omega}} \left(\hat{\rho} - \frac{3\hat{p}}{c^2} \right) + \frac{cc'}{4\pi G \hat{b}^2} , \quad (12)$$

and

$$\hat{\rho}'_\sigma + \left(\frac{\hat{a}'}{\hat{a}} + 2\frac{\hat{b}'}{\hat{b}} \right) \left(\hat{\rho}_\sigma + \frac{\hat{p}_\sigma}{c^2} \right) = \frac{\sigma'}{\sqrt{6+4\omega}} \left(\hat{\rho} - \frac{3\hat{p}}{c^2} \right) \quad (13)$$

Thus standard KS model field equations with constant G for VSL are also the field equations for BD theory in Einstein frame but here one must add to normal matter the scalar field energy and pressure . Therefore , the total energy density and pressure are given by $\hat{\rho}_t = \hat{\rho} + \hat{\rho}_\sigma$ and $\hat{p}_t = \hat{p} + \hat{p}_\sigma$ respectively .

III. PERTURBATIVE SOLUTIONS TO THE FLATNESS PROBLEM

For a possible solution to the flatness problem in VSL theory , let us first study solutions when there are small deviations from flatness . The critical energy density ρ_c in a BD universe is given by the equation

$$\left(\frac{\dot{b}}{b} \right)^2 + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} = \frac{8\pi\rho_c}{\phi} - \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) \frac{\dot{\phi}}{\phi} + \frac{\omega}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 \quad (14)$$

In Einstein frame the critical energy density for normal matter is

$$\hat{\rho}_c = \frac{1}{8\pi G} \left[\left(\frac{\hat{b}}{\hat{b}} \right)^2 + 2\frac{\hat{a}'}{\hat{a}}\frac{\hat{b}'}{\hat{b}} - \frac{\hat{\sigma}'^2}{2} \right] = \frac{\rho_c}{G^2\phi^2} \quad (15)$$

Hence,if we define the total critical energy as

$$\hat{p}_\alpha = \hat{\rho}_c + \hat{\rho}_\sigma ,$$

then from (15) we have

$$\hat{\rho}_\alpha = \frac{1}{8\pi G} \left[\left(\frac{\dot{b}}{\hat{b}} \right)^2 + 2 \frac{\hat{a}'}{\hat{a}} \frac{\hat{b}'}{\hat{b}} \right] \quad (16)$$

Let us define a relative flatness parameter as

$$\varepsilon_t = \frac{\hat{\rho}_t - \hat{\rho}_\alpha}{\hat{\rho}_\alpha} \quad (17)$$

whose evolution equation is given by

$$\hat{\varepsilon}'_t = (1 + \hat{\varepsilon}_t) \hat{\varepsilon}_t \left(\gamma \frac{\hat{a}'}{\hat{a}} + 2(\gamma - 1) \frac{\hat{b}'}{\hat{b}} \right) + 2 \frac{c'}{c} \hat{\varepsilon}_t \quad (18)$$

Here $\gamma - 1 = p/\rho c^2 = \dot{p}/\dot{\rho}c^2$, is the equation of state, with γ constant. Since

$$\hat{\varepsilon}'_t = \frac{\varepsilon}{1 + (2\omega + 3)\dot{\phi}^2/32\pi\phi\rho_c} \quad (19)$$

in Jordan frame, the deviation from flatness is given by

$$\delta = \frac{\rho - \rho_c}{\rho_c + (2\omega + 3)\dot{\phi}^2/32\pi\phi\rho_c} < \varepsilon \quad (20)$$

where $\varepsilon = (\rho - \rho_c)/\rho_c$ measures natural deviations from flatness. Now δ , the adaptation parameter satisfies the differential equation

$$\dot{\delta} = \delta(1 + \delta) \left(\gamma \frac{\dot{a}}{a} + 2(\gamma - 1) \frac{\dot{b}}{b} + \frac{1}{2}(3\gamma - 2) \frac{\dot{\phi}}{\phi} \right) + 2 \frac{\dot{c}}{c} \delta \quad (21)$$

If δ is assumed to be very small compare to unity i.e., $\delta \ll 1$, then neglecting the square term in (21), one can integrate the above to get

$$\delta = \delta_0 a^\gamma b^{2(\gamma-1)} \phi^{(3\gamma-2)/2} c^2 \quad (22)$$

where δ_0 is the integration constant. Hence we have

$$\frac{1}{\varepsilon} = \frac{\varepsilon_0}{a^\gamma b^{2(\gamma-1)} \phi^{(3\gamma-2)/2} c^2} - \frac{(2\omega + 3)\dot{\phi}^2}{32\pi\rho_c\varepsilon} \quad (23)$$

In BD theory $G \propto \phi^{-1}$, so to solve the flatness problem ϕ should decrease in the early universe and the first term on the right hand side must dominate over the second one.

In particular, in radiation era ($\gamma = 4/3$) the above equation (23) simplifies to

$$\frac{1}{\varepsilon} = \frac{\varepsilon_0}{a^{4/3} b^{2/3} \phi c^2} - \frac{(2\omega + 3)\alpha^2}{32\pi\phi a^2 b^4 \rho_c \varepsilon} \quad (24)$$

where we have used equation (5) to obtain ϕ . If α is small and $c = c_0 [a^n b^{2(n-1)}]^{n/(3n-2)}$ then [12,3] it is still possible to solve the flatness problem for $n < -1 - \sqrt{7/3}$. Furthermore, since $\rho_c \propto 1/(ab^2)^{4/3}$ at late times, so the second term is always very small compare to the first and hence may be neglected.

IV. NON-PERTURBATIVE SOLUTIONS WITH $\phi = \text{CONSTANT}$: MATTER AND RADIATION DOMINATED ERA

When $\phi = \phi_0$ (constant) we have $\delta = \varepsilon$ in (20) and the differential equation for δ , (21) can be written as

$$\dot{\varepsilon} = \varepsilon(1 + \varepsilon) \left(\gamma \frac{\dot{a}}{a} + 2(\gamma - 1) \frac{\dot{b}}{b} \right) + 2 \frac{\dot{c}}{c} \varepsilon \quad (25)$$

Now, if we assume a power-law form in the scale factors of the velocity of light then the above equation becomes

$$\dot{\varepsilon} = \varepsilon(1 + \varepsilon) \left(\gamma \frac{\dot{a}}{a} + 2(\gamma - 1) \frac{\dot{b}}{b} \right) + 2\varepsilon \left(\frac{n}{3n-2} \right) \left(n \frac{\dot{a}}{a} + 2(n-1) \frac{\dot{b}}{b} \right) \quad (26)$$

In order to get an exact analytic solution to the above equation, we assume $\varepsilon \ll 1$, so we can neglect ε^2 -term. We then have the solution

$$\varepsilon = \varepsilon_0 a^{\gamma+2n^2/(3n-2)} b^{2(\gamma-1)+4n(n-1)/(3n-2)} \quad (27)$$

with ε_0 as the integration constant.

Furthermore, without assuming any restriction on the parameter ε , if we assume the two arbitrary constants n and γ to be equal then also we have an exact analytic solution for ε ,

$$\varepsilon = -\frac{1}{\frac{3\gamma-2}{5\gamma-2} + A [a^\gamma b^{2(\gamma-1)}]^{-(5\gamma-2)/(3\gamma-2)}} \quad (28)$$

where A is an integration constant.

V. EXACT SOLUTIONS TO THE FLATNESS PROBLEM: RADIATION DOMINATED ERA

To obtain the exact solutions to the flatness problem we assume the variation of light mathematically by the relation [3,12]

$$c = c_0 \left[\left\{ a^n b^{2(n-1)} \right\}^{1/(3n-2)} \sqrt{\phi G} \right]^n \quad (29)$$

However, in the Einstein frame the above relation reduces to $c = c_0 \left[\hat{a}^n \hat{b}^{2(n-1)} \right]^{1/(3n-2)}$ and equation (12) becomes

$$\hat{\rho}' + \left(\frac{\hat{a}'}{\hat{a}} + 2 \frac{\hat{b}'}{\hat{b}} \right) \left(\hat{\rho} + \frac{\hat{p}}{c^2} \right) = \frac{cc'}{4\pi G \hat{b}^2} \quad (30)$$

The equation of state $\hat{p} = (\gamma - 1)\hat{\rho}c^2$ simplifies the above equation further to

$$\hat{\rho}' + \gamma \hat{\rho} \left(\frac{\hat{a}'}{\hat{a}} + 2 \frac{\hat{b}'}{\hat{b}} \right) = \frac{cc'}{4\pi G \hat{b}^2} \quad (31)$$

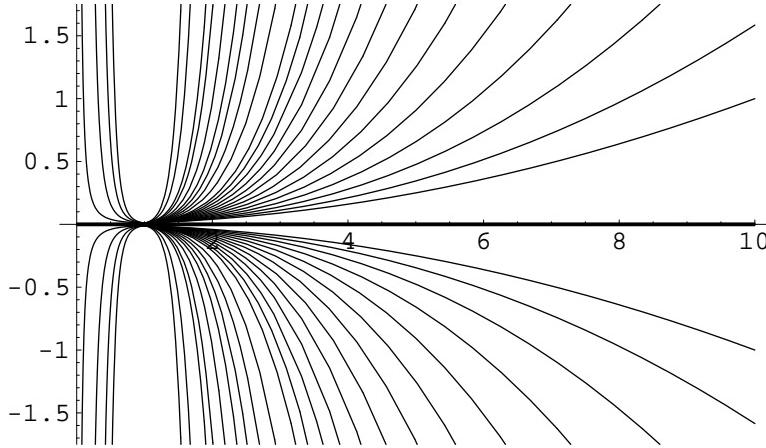


FIG. 1: Here the flatness parameter ε in (27) has been plotted against t for the radiation era ($\gamma = 4/3$) choosing different values for the constant n . The scale factors here are assumed to be in simple expanding form.

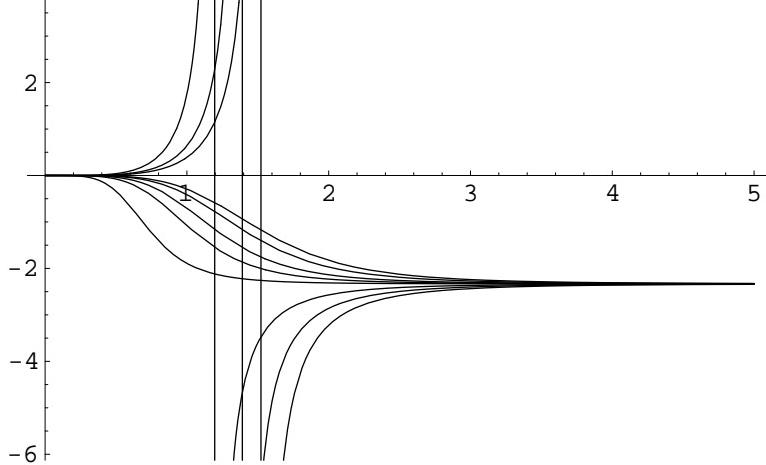


FIG. 2: This figure shows the variation of the flatness parameter from (28), where there is no restriction on ε . This figure is also drawn for the radiation era and assuming as before the simple expanding form for the scale factors. The variation of ε for different choice of the arbitrary constant A is shown in the figure.

An exact integral can be obtained (assuming $n = \gamma$) as

$$\hat{\rho}'(\hat{a}\hat{b}^2)^\gamma = B + \frac{\gamma c_0^2}{4\pi G(5\gamma - 2)} \left[\hat{a}^\gamma \hat{b}^{2(\gamma-1)} \right]^{(5\gamma-2)/(3\gamma-2)} \quad (32)$$

Hence for the radiation dominated era , we have in Jordan frame

$$\rho(ab^2)^{4/3} = B + \frac{c_0^2}{14\pi G} (a^2 b)^{14/9} (\phi G)^{7/3} \quad (33)$$

where we have used $\hat{\rho}(\hat{a}\hat{b}^2)^{4/3} = \rho(ab^2)^{4/3}$.

Moreover, in the Jordan frame we have equation (29) for the velocity of light and we can write

$$\frac{\dot{c}}{c} = \frac{n}{3n-2} \left[n \frac{\dot{a}}{a} + 2(n-1) \frac{\dot{b}}{b} \right] + \frac{n}{2} \frac{\dot{\phi}}{\phi} \quad (34)$$

Thus, if ϕ is a decreasing function then $|\dot{c}/c| < |[n\dot{a}/a + 2(n-1)\dot{b}/b]/(3n-2)|$ and we can therefore conclude that a varying speed of light in the early universe (with weaker gravity) helps to solve the flatness problem.

Alternatively, for general equation of state $p = (\gamma - 1)\rho c^2$, if we make the transformation

$$x = \phi a^\gamma b^{2(\gamma-1)} \quad (35)$$

and assume the variation of the velocity of light as

$$c = c_0 x^{n/2}$$

then the equation of continuity (4) becomes

$$\dot{\rho} + \gamma \rho \left(\frac{\dot{a}}{a} + 2 \frac{\dot{b}}{b} \right) = \frac{n c_0^2 \phi x^{n-1} \dot{x}}{8\pi b^2} \quad (36)$$

Integrating this equation gives

$$\rho a^\gamma b^{2\gamma} = B + \frac{n c_0^2 \phi^{n+1} [a^\gamma b^{2(\gamma-1)}]^{n+1}}{8\pi(n+1)} \quad (37)$$

(provided $n \neq -1$) which gives the evolution for general γ .

VI. SOLUTIONS TO THE QUASI-FLATNESS PROBLEM

A. When BD scalar field ϕ is constant

In this section we shall investigate whether it is natural to have evolution which asymptotes to a state of expansion with a non-critical density. Let us start with the continuity equation for constant ϕ and assume as before the equation of state

$$\frac{p}{\rho c^2} = \gamma - 1 ,$$

and the velocity of light to be

$$c = c_0 \left[a^n b^{2(n-1)} \right]^{n/(3n-2)}$$

then the integral gives

$$\rho (ab^2)^\gamma = B + \frac{\gamma c_0^2 \phi}{4\pi(5\gamma-2)} \left[a^\gamma b^{2(\gamma-1)} \right]^{(5\gamma-2)/(3\gamma-2)} \quad (38)$$

(It is to be noted that in order to obtain the integral one has to assume $n = \gamma$).

Now substituting this value of ρ in equation (2) we have

$$\begin{aligned} \left(\frac{\dot{b}}{b}\right)^2 + 2\frac{\dot{a}\dot{b}}{ab} &= \frac{8\pi}{\phi} - \frac{B}{(ab^2)^\gamma} + \frac{\gamma c_0^2 \phi}{4\pi(5\gamma-2)} + \frac{2[a^\gamma b^{2(\gamma-1)}]^{(5\gamma-2)/(3\gamma-2)}}{(ab^2)^\gamma} \\ &\quad - \frac{1}{b^2} c_0^2 \left[a^\gamma b^{2(\gamma-1)}\right]^{2\gamma/(3\gamma-2)} \end{aligned} \quad (39)$$

In particular, for radiation era the above equation simplifies to

$$\left(\frac{\dot{b}}{b}\right)^2 + 2\frac{\dot{a}\dot{b}}{ab} = \frac{8\pi B}{\phi(ab^2)^{4/3}} - \frac{3}{7} c_0^2 (a^8 b^{-5})^{2/9} \quad (40)$$

If Ω is the density parameter, then it can be defined as

$$\frac{\Omega}{\Omega-1} = \frac{8\pi G\rho}{c^2/b^2} \quad (41)$$

We note that the ratio between the two terms on the right hand side is almost a constant for a quasi-flat open universe. However, for $\gamma = 4/3$, we get

$$\frac{\Omega}{\Omega-1} = \frac{4}{7} + \frac{8\pi GB}{c_0^2(a^2 b)^{14/9}} \quad (42)$$

For expanding universe a, b grows with time. So asymptotically the second term on the R.H.S will be negligible and we have

$$\Omega = -\frac{4}{3}$$

which leads to an open universe with finite Ω value today.

B. When BD scalar field varies

For the radiation era ($\gamma = 4/3$), let us define

$$y = \phi(ab^2)^{2/3}, \quad z = \phi(a^2 b)^{2/3}, \quad c = c_0 z^m \quad (43)$$

So from the field equation (2) we get (using equation (5))

$$2\frac{y'}{y}\frac{z'}{z} - \frac{z'^2}{z^2} = \frac{32\pi\rho}{3y}(ab^2)^{4/3} - \frac{4c^2}{3}\frac{z}{y} + \left(1 + \frac{2\omega}{3}\right)\frac{\alpha^2}{y^2} \quad (44)$$

Now, from equation (38) the expression of ρ takes the form

$$\rho(ab^2)^{4/3} = \frac{Bmc_0^2 z^{2m+1}}{4\pi(2m+1)} \quad (45)$$

In addition the critical density ρ_c is obtained from the differential equation

$$2\frac{y'}{y}\frac{z'}{z} - \frac{z'^2}{z^2} = \frac{32\pi\rho_c}{3y}(ab^2)^{4/3} + \left(1 + \frac{2\omega}{3}\right)\frac{\alpha^2}{y^2} \quad (46)$$

Hence the expression for the density parameter is

$$\Omega = \frac{\rho}{\rho_c} = \frac{2\frac{y'z'}{y} + \frac{4c^2}{3}\frac{z}{y} - \left(1 + \frac{2\omega}{3}\right)\frac{\alpha^2}{y^2}}{2\frac{y'z'}{y} - \frac{z'^2}{z^2} - \left(1 + \frac{2\omega}{3}\right)\frac{\alpha^2}{y^2}}$$

or equivalently,

$$\frac{\Omega}{\Omega - 1} = \frac{\rho}{\rho - \rho_c} = \frac{8\pi B}{c_0^2 z^{2m+1}} + \frac{2m}{2m + 1} \quad (47)$$

Thus, if $2m + 1 < 0$ then as $t \rightarrow \infty, z \rightarrow \infty$ so we have $\Omega \rightarrow 1$. But if $2m + 1 > 0$ then $t \rightarrow \infty, \Omega \rightarrow -2m$. So asymptotically, we have a quasi-flat open universe.

C. General asymptotic behaviour

In this section, we consider general asymptotic behaviour when $2m + 1 > 0$ i.e., for quasi-flat open universe. The equation (44) can be approximated by

$$2y'z' - \frac{yz'^2}{z} \simeq \Gamma z^{2(m+1)}$$

which has a first integral

$$\frac{y}{\sqrt{z}} = \int \frac{\Gamma z^{2(m+1)}}{2z'\sqrt{z}} d\tau$$

Thus, if we assume $z \sim \tau^{2/\Omega_\infty}$, then $y \sim \tau^{2/\Omega_\infty}$ and

$$\phi \sim \phi_0 \exp\left(\frac{\tau^{1-2/\Omega_\infty}}{1 - \tau^{2/\Omega_\infty}}\right) \quad (48)$$

For the case of $\Omega_\infty < 1$, we have $\phi \rightarrow \phi_0$, $a(\tau) \sim \tau^{1/\Omega_\infty}$ and $b(\tau) \sim \tau^{1/\Omega_\infty}$ in the limit as $\tau \rightarrow \infty$, or equivalently,

$$\phi \rightarrow \phi_0, \quad a \sim \tau^{1/(\Omega_\infty+1)}, \quad b \sim \tau^{1/(\Omega_\infty+1)}, \quad \text{as } t \rightarrow \infty.$$

In addition, when $\Omega_\infty = 1$, we get $a \sim t^{1/2}$, $b \sim t^{1/2}$, as expected for flat radiation asymptote .

VII. THE LAMBDA AND THE QUASI-LAMBDA PROBLEMS

In this section, we shall examine the effect of variation of speed of light when a cosmological term is introduced in the BD field equations. In fact, the incorporation of a cosmological term is equivalent to introduction of a vacuum stress which obeys an equation of state

$$\rho_\Lambda = -\frac{p_\Lambda}{c^2}$$

with

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$$

Thus equation of continuity can be generalized to

$$(\dot{\rho} + \dot{\rho}_\Lambda) + \left(\frac{\dot{a}}{a} + 2 \frac{\dot{b}}{b} \right) = \frac{c\dot{c}}{4\pi G^2 b}$$

Also the field equation (2) can now be written as

$$\frac{\dot{b}^2}{b^2} + 2 \frac{\dot{a}}{a} \frac{\dot{b}}{b} = 8\pi G\rho - \frac{c^2}{b^2} + \Lambda c^2 \quad (49)$$

Now, using the above expression for vacuum stress-energy and using $\gamma - 1 = p/\rho c^2$ as the equation of state for the matter and assuming

$$c = c_0 \left[a^n b^{2(n-1)} \right]^{n/(3n-2)},$$

the above equation of continuity can be integrated (assuming $n = \gamma$) to give

$$\rho = \frac{B}{(ab^2)^\gamma} + \frac{\gamma c_0^2}{8\pi G(5\gamma - 2)} \frac{[a^\gamma b^{2(\gamma-1)}]^{\frac{5\gamma-2}{3\gamma-2}}}{(ab^2)^\gamma} - \frac{\Lambda c_0^2(5\gamma - 2)}{8\pi G} \frac{1}{ab^2} \int (ab^2)^\gamma d[a^\gamma b^{2(\gamma-1)}]^{\frac{2\gamma}{3\gamma-2}} \quad (50)$$

On substituting this value of ρ in the above field equation (49), we have

$$\begin{aligned} \frac{\dot{b}^2}{b^2} + 2 \frac{\dot{a}}{a} \frac{\dot{b}}{b} &= \frac{8\pi GB}{(ab^2)^\gamma} + \frac{2\gamma c_0^2}{(5\gamma - 2)} \frac{[a^\gamma b^{2(\gamma-1)}]^{(5\gamma-2)/(3\gamma-2)}}{(ab^2)^\gamma} + \left(\Lambda - \frac{1}{b^2} \right) c_0^2 \left[a^\gamma b^{2(\gamma-1)} \right]^{2\gamma/(3\gamma-2)} \\ &\quad - \Lambda c_0^2(5\gamma - 2) \frac{1}{(ab^2)^\gamma} \int (ab^2)^\gamma d[a^\gamma b^{2(\gamma-1)}]^{2\gamma/(3\gamma-2)} \end{aligned} \quad (51)$$

If we define the generalized density parameter as

$$\Omega_1 = \Omega_m + \Omega_\Lambda = \frac{8\pi G(\rho + \rho_\Lambda)b^2}{c^2} \quad (52)$$

then using the density parameter Ω from equation (41) we have

$$\Omega_1 = \frac{\Omega}{\Omega - 1} = \frac{8\pi G(\rho + \rho_\Lambda)b^2}{c^2}$$

If we now substitute the value of ρ from equation (50) we obtain

$$\begin{aligned} \frac{\Omega}{\Omega - 1} &= \frac{2\gamma}{5\gamma - 2} \left[a^\gamma b^{2(\gamma-1)} \right]^{(4\gamma-3)/(3\gamma-2)} + \frac{8\pi GB}{c_0^2} \left[a^\gamma b^{2(\gamma-1)} \right]^{-(4\gamma-2)/(3\gamma-2)} + \Lambda b^2 \\ &\quad - \Lambda(3\gamma - 2) \left[a^\gamma b^{2(\gamma-1)} \right]^{-(4\gamma-2)/(3\gamma-2)} \int (ab^2)^\gamma d[a^\gamma b^{2(\gamma-1)}]^{2\gamma/(3\gamma-2)} \end{aligned} \quad (53)$$

when $\gamma > 1$. The 2nd, 3rd and 4th terms are negligible compare to the first term, hence we have

$$\frac{\Omega}{\Omega - 1} \sim \frac{2\gamma}{5\gamma - 2} \left[a^\gamma b^{2(\gamma-1)} \right]^{(4\gamma-3)/(3\gamma-2)} \quad (54)$$

So, for large a, b (i.e., $a, b \rightarrow \infty$)

$$\frac{\Omega}{\Omega - 1} \rightarrow \infty \quad i.e., \quad \Omega \rightarrow 1.$$

Furthermore, if $\Lambda = 0$ then

$$\frac{\Omega}{\Omega - 1} = \frac{2\gamma}{5\gamma - 2} [a^\gamma b^{2(\gamma-1)}]^{(4\gamma-3)/(3\gamma-2)} + \frac{8\pi GB}{c_0^2} [a^\gamma b^{2(\gamma-1)}]^{-(4\gamma-2)/(3\gamma-2)} \quad (55)$$

So for the radiation era ($\gamma = 4/3$)

$$\frac{\Omega}{\Omega - 1} = \frac{8}{14} + \frac{8\pi GB}{c_0^2} (a^2 b)^{-14/9}. \quad (56)$$

As $a, b \rightarrow \infty$, $\Omega/(\Omega - 1) \rightarrow 8/14$ i.e., $\Omega = -4/3$. This asymptotic behaviour is same as in Sec.VIA where for quasi-flatness problem the BD scalar field is assumed to be constant.

VIII. DISCUSSION

In this work an extensive analysis of the BD solutions for anisotropic cosmological model has been done where there is a variation of the velocity of light with time we have assumed the velocity of light to be in power-law form in the scale factors. The flatness problem has been discussed in details. Here perturbative, non-perturbative and exact solutions of flatness problem have been obtained. The graphical representation for non-perturbative solution in the radiation era has some interesting feature. Quasi-flatness problem with general asymptotic behaviour has also been discussed and in most cases we obtain an open universe. Finally, we have also studied the lambda and quasi-lambda problem in VSL and the asymptotic behaviour is very similar to that in the flatness problem.

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